The Cycloid Permanent Magnetic Gear
Frank T. Jørgensen, Torben Ole Andersen, and Peter Omand Rasmussen

Abstract—This paper presents a new permanent-magnet gear based on the cycloid gearing principle, which normally is characterized by an extremely high torque density and a very high gearing ratio. An initial design of the proposed magnetic gear was designed, analyzed, and optimized with an analytical model regarding torque density. The results were promising as compared to other high-performance magnetic-gear designs. A test model was constructed to verify the analytical model.

Index Terms—Analytical modeling, finite element analysis (FEA), magnetic gear, permanent magnets.

I. INTRODUCTION

RECENTLY, magnetic gears have gained some attention due to the following reasons: no mechanical fatigue, no lubrication, overload protection, reasonably high torque density, and potential for very high efficiency. Focus [1], [4], [5] have been addressed to a kind of planetary magnetic gear, probably already invented before the strong NdFeB magnets came into the market in the early 1980s [6]. An active torque density is in the range of 100 N·m/L, which is a very high torque density for a magnetic device [4]. However, there is still a need for increased torque density and a better utilization of the permanent magnets. The torque density of a magnetic coupling is in the range of 400 N·m/L, and this is, in principle, a magnetic gear with a 1:1 gearing ratio.

In this paper, a magnetic gearing topology with better utilization of the permanent magnet is presented. This gearing topology makes it possible to increase the torque density to almost twice the state-of-the-art magnetic gears and, therefore, might be a useful alternative in applications using traditional mechanical gears or, at least, in gearing applications where some of the other advantages, e.g., overload protection, oil-free construction, and separation, are vital.

This paper will first give a description of the cycloid permanent magnetic gear and how the idea is derived from the classical magnetic spur gear. Due to the fact that the cycloid permanent magnetic gear is a 2-DOF topology, description of gearing ratios with different fixed axes (1 DOF) is stated. In order to optimize the layout of the new cycloid magnetic gear, a parametric analytical model to calculate the torque density is developed.

The construction of an initial test model is given, and measurement is performed in order to validate the analytical model. Next, an optimization is utilized with the analytical model to quantify the cycloid permanent-magnet gear capability, and finally, a conclusion is given.

II. DESCRIPTION AND DEVIATION OF THE CYCLOID PERMANENT MAGNETIC GEAR

A permanent-magnet version to the classical spur gear can be made, where the teeth are substituted by permanent magnets; see Fig. 1.

From Fig. 1, it is quite clear that a lot of the magnets are inactive and cannot assist in transferring torque between the two rings. In addition, volume taken up from the gear is quite high because the two rings are separated. In order to reduce the volume and also increase the interaction, it is therefore more suitable to use an inner type spur gear; see Fig. 2(a).

From Fig. 2, it is shown that more magnets gets active if the number of poles on the inner ring is getting closer to the number of poles on the outer ring. If the number of poles on
the inner ring is equal to the number of poles on the outer ring, the gearing ratio will be unity, and the gear can be considered as a magnetic coupling. With almost equal number of poles on the two rings, the gearing ratio is very low. Fig. 2(b) shows an example where the number of poles on the outer rotor is 44 and on the inner rotor 42. This example gear has very high magnetic interaction but the gearing ratio is only $\frac{44}{42} \approx 1.05$.

In order to improve the low gearing ratio for the gear shown in Fig. 2(b), a cycloidal principle is considered. Cycloidal gearing principle has significant gear-reduction capability. Movement principle for this gear is explained with nine illustrations shown in Fig. 3. The illustrations show an outer rotor with 44 poles and an inner rotor with 42 poles. The outer rotor is stationary while the inner rotor is magnetically connected to outer rotor and placed eccentrically relative to outer rotor. Clockwise angular-position change for the air gap will result in a small change of the inner rotor rotation in anticlockwise direction. The angular position for the air gap has rotated one anticlockwise revolution from illustrations one to nine while the inner rotor has only rotated $\frac{1}{21}$ revolution anticlockwise.

This gearing principle has significant gear-reduction ability. Same example is shown in Fig. 4(c), where the outer rotor part C is fixed. An eccentric B is driving the inner magnetic plate, and this plate will make a combined orbit and rotational motion. The rotational part of this motion is transferred to the output shaft A. The gear ratio is $(-21/1) = -21$, which is much higher than a simple inner spur-gear configuration.

### III. GEARING RELATIONSHIP FOR THE CYCLOIDAL GEAR

The movement principle of the permanent-magnet cycloid gear is described in the previous section, where the outer rotor is fixed from rotation. However, it is also possible to fix other parts of the gear. In Fig. 4, the other possible combinations of fixed parts are shown together with equations for the gearing ratio.

Gear configuration in Fig. 4(c) is the configuration already described in Section II, and the gear configuration in Fig. 4(b) is equivalent to the internal spur gear with the relatively low gearing ratio. The configuration shown on Fig. 4(a) has the largest gearing ratio, and the input and output axes are separated by an air gap. The configuration in Fig. 4(c) has similar characteristics as the one in Fig. 4(a) and may be preferred in some applications due to its layout for integration. The torque density is almost similar for all configurations.

### IV. ANALYTICAL MODEL

In order to be able to design and optimize the cycloid magnetic gear, a model is required. An analytical model is preferred when optimization has to be applied because of significant reduced computational time as compared to a finite element analysis (FEA).

Due to the fact that the permanent-magnet cycloid gear is a variation of the permanent-magnet spur gear, it is obvious to use the same theory used for this gear type. In [2], the authors have derived an analytical model for permanent magnetic spur gears with parallel magnetized magnets. This model is only briefly introduced as function expressions in this paper, and for further detailed explanation, the reader referred to [2] and [3].

#### A. Magnetic-Field Expression

The magnetic-field solutions for a parallel magnetized magnetic ring are written by (1) and (2). This magnetic ring is the so-called source shown in Fig. 5(a).

Equations (1) and (2) have origin in the source coordinate system and, therefore, named $B_i^{(2)}(r', \phi', v)$ and $B_o^{(2)}(r', \phi', v)$, respectively,

$$B_{i}^{(2)}(r', \phi', v) = \mu_0 \sum_{i=1,3,5,...}^{\infty} \frac{1}{2} i N_p r'^{-\left(4 N_p + i\right)} \times U(2) \left(\frac{1}{2} N_p \phi'\right)$$

$$B_{o}^{(2)}(r', \phi', v) = \mu_0 \sum_{i=1,3,5,...}^{\infty} \frac{1}{2} i N_p r'^{-\left(4 N_p + i\right)} \times U(2) \left(\frac{1}{2} N_p \phi'\right)$$
These field-solution equations are transformed into the drive-magnet coordinate system $B^{\text{ext}}(r, \theta, \phi, v)$ and $B_{\phi}^{\text{ext}}(r, \theta, \phi, v)$. Further explanation of the field transformation is explained in [3]. The two field expressions depend on coefficient terms, and these terms can be expressed as (3)–(8). The field expressions (1) and (2) are indirectly used in parts of the torque calculation expression (9)

\[
U_i^{(2)}(v) = -2\mu_0 \frac{H_{ri}(v)}{L_i(v)} M_{ri} - 2\mu_0 \frac{H_{phi}(v)}{L_i(v)} M_{phii} \tag{3}
\]

\[
H_{ri}(v) = R_{2s}^2 \mu_0 \left( R_{2s}^{N_{pi}} \right)^{\frac{3}{2}} N_{pi} + 2 R_{2s}^2 \mu_0 R_{1s}^2 \sqrt{R_{2s}^{N_{pi}} R_{2s}^2} + R_{1s}^2 \mu_0 R_{1s}^2 \right) \right)
\]
### TABLE I Dimensions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Dimensions for analytical model</th>
<th>Description</th>
<th>Size</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_{CyC}</td>
<td>Cycloid gear torque Figure 3c.</td>
<td>43.91</td>
<td>Nm</td>
<td></td>
</tr>
<tr>
<td>T_{in}</td>
<td>Spur gear torque [2], Figure 3b.</td>
<td>46</td>
<td>Nm</td>
<td></td>
</tr>
<tr>
<td>R_{2s}</td>
<td>Outer radius, source magnets(^a)</td>
<td>53.5</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>R_{1s}</td>
<td>Inner radius, source magnets</td>
<td>48.5</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>R_{2d}</td>
<td>Outer radius, drive magnets</td>
<td>61.5</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>R_{1d}</td>
<td>Inner radius, drive magnets</td>
<td>56.5</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Distance between center points</td>
<td>2.5</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>Vector containing opt. variables</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Length/height of the magnets</td>
<td>26</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>B_r</td>
<td>Remanence flux</td>
<td>1.21</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>H_c = M_k</td>
<td>Coercivity</td>
<td>915000</td>
<td>A/m</td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>Harmonic parameter, max value.</td>
<td>4</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>N_{i} = N_{t}</td>
<td>Torque integration parameter</td>
<td>10</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>\mu_0</td>
<td>Permeability for air regions</td>
<td>4\pi \times 10^{-7}</td>
<td>Tm/A</td>
<td></td>
</tr>
<tr>
<td>\mu = B_r/H_c</td>
<td>Permeability, magnets</td>
<td>13.22 \times 10^{-7}</td>
<td>Tm/A</td>
<td></td>
</tr>
<tr>
<td>\mu_r = \mu/\mu_0</td>
<td>Relative permeability, magnets</td>
<td>1.0523</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>N_p = P_A</td>
<td>Number of source pole magnets</td>
<td>42</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>N_{pole} = P_C</td>
<td>Number of drive pole magnets</td>
<td>44</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>\phi</td>
<td>Max. torque calculation angle</td>
<td>4.09</td>
<td>Deg.</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) The index s and b are used for source and drive magnets. (Figure 4)

C. Modified Torque Calculation From Spur Gear to Cycloid Type

The torque-calculation model for the spur gear is general, and thereby, it is possible to change the separation distance between the two rings to the eccentricity of the inner ring, see Fig. 5. This dimensional change will only change the equations for flux transformations [3]. Source and drive magnet is inside of each other, and the torque is calculated on the outer drive magnet (9). The system configuration is equivalent to the inner magnetic spur gear shown in Fig. 2(b).

Torque equations, for the gear type shown in Fig. 4(b), can be used to calculate the nominal corresponding output torque of a gear type shown in Fig. 4(c). This torque transformation can be written by

$$T_{CyC} = T_{in} \cdot \frac{P_A}{P_C}. \quad (16)$$

Pole-number configuration for the cycloid permanent-magnet initial design comes partly from a previous developed magnetic gear [1]. The gear in [1] has 44 permanent magnets on the outer ring, and this pole number is reused for the cycloid permanent-magnet gear design. The inner ring is designed with 42 magnets. The minimum air gap is fixed at 0.5 mm, and the eccentricity is optimized to be 2.5 mm. The parameters for the initial design are listed in Table I.

In Table I, it is seen that the initial cycloid permanent-magnet gear has a calculated torque density of 141.9 N·m/L, i.e., around 40% more than the “planetary” magnetic gears developed in [4]. In order to validate the analytical model for the initial design, a static FEA was made, and the results from the two models are shown in Fig. 6.

In Fig. 6, good agreement is seen between the two calculation methods. The reason for the small deviation is mainly caused by the assumption of a unity relative permeability of the permanent magnets in the derivation of the analytical model. The FEA model uses 1.05 in relative permeability for the NdFeB magnets.

V. Construction of a Test Model

Starting point for making a test model is the cycloidal moving principle shown in Fig. 3. It was chosen to use sheet steel yokes for both rings, meaning that the height of the permanent magnets can be reduced by a factor of two because the steel will bridge neighboring magnets (magnetic mirrors). The steel yoke has also the advantages of limiting magnetic field around the construction.

In order to make the test model simple, rectangular magnets were used instead of arc-shaped magnets, and to reduce large unbalanced magnetic forces, two gear sets were used. Fig. 7 shows a rendered view of the constructed model. Outer dimensions of the experimental test model are 130 × 130 mm, and the gearbox total length is 86 mm.

The input shaft is placed at the center of the gearbox. A bearing is placed on the eccentric and connected to the inner steel yoke. Circular motion of the input shaft will force the inner yoke to move in orbital motion. Inner yoke magnets will perform a cycloidal motion. Strong magnetic-flux paths between the inner and outer ring will ensure torque on the output shaft. The output shaft is connected to the inner yoke with tree columns and eccentrics. These eccentrics can compensate for misalignment between inner yoke and output shaft. The columns will act like mechanical coupling and transfer torque from the inner yoke to the output shaft.

The rotor yokes are placed on the eccentric input shaft. The eccentricity is made by a central eccentric bushing. Together with six support eccentrics, 12 extra needle bearings are added.
Their functionality is to ensure a parallel motion of the inner rotors relative to the output shaft reference. The proposed construction will therefore have 18 bearings in total.

The total volume of the magnetic gear could have been reduced by choosing a circular design and also by choosing smaller bearings.

The cycloid gear has been tested in two situations. The first test was a static test where the output torque was measured to be 33 N·m. This measurement is shown in Fig. 6 with a single measurement at peek output torque for comparison with the 2-D models. The measured torque of the experimental test model is lower than the results from the 2-D models. For this deviation, 3-D end effects are assumed to be the most dominant factor.

It has to be noted that the length of the magnets are very small, so large 3-D effects should be expected. The length of the magnet is based on the availability of standard magnets and, thus, not optimized. However, for future optimization, it might be worth to consider the third dimension in the analytical calculation methods, since it apparently has a large effect for this relatively short layout. Basically, this means that the stack length cannot be considered as a simple scaling parameter as it is for typical electrical machines.

The constructed cycloid gear is reasonably comparable with the gear developed in [1]. Both gears use 44 times two of the same type of rectangular magnets in the outer ring, and the air-gap radius is, thus, similar. In [1], 216 standard rectangular magnets were used, while the cycloid gear in this paper only uses 172 of the same type of magnet. In [1], the stall torque was measured to 16 N·m, which means that the cycloid gear has doubled the torque density.

The other test was a dynamic test where input and output torques were measured. The test was performed with two servo drives. One of the drives was set up as load drive, and the other was set up as input drive. Three input velocities were tested, and different loads were applied by the load control drive. The torque on both sides of the gear was measured with torque transducers (Fig. 8).

Efficiencies were calculated from the measured torque values, and the results are shown in Fig. 9.

The efficiency were measured at 1500, 500, and 50 r/min. The highest efficiencies were generally obtained at low speed and high torque. The best gear efficiency measured at 50 r/min was 94%. Efficiencies of 500 and 1500 r/min were 93% and 92%, respectively.

VI. OPTIMIZATION

The initial design may not give the full picture of the capability of the cycloid permanent-magnet gear. Optimizations were therefore performed by formulating the analytical equations to a general optimization problem [7] with a cost function (17), equality constraints (18), inequality constraints (19), and a number of constants from Table I. The cost function (17) is the torque density of the magnetic spur gear with the torque calculated from the parallel magnetization expressions developed in [2]. The volume is set by the area given by drive-magnet wheel radius and a constant length. It is necessary to have an equality constrain $h_1$ for limitation of the used permanent-magnet material, i.e., the area, since the length is fixed. This permanent-magnet area constrain were set up to be the same area as used for the analytical model of the initial magnetic-gear test model ($A_{\text{const}} = 35.4 \cdot 10^{-4} \text{m}^2$). Another equality constrain $h_2$ is set to keep a separation distance of 0.5 mm between the magnetic wheels. Inequality constrains is set for the minimization routine. The outer radius has to be somewhat greater than the inner radius in $g_1$ and $g_2$. Largest radius of the drive wheel radius has to be less than a certain value in $g_3$. The smallest radius for the inner gear wheel had to be greater than
a certain value in \( g_4 \). The distance of the eccentric also has a limitation in \( g_5 \) and \( g_6 \)

\[
f(x) = f(x_1, x_2, \ldots, x_n) \Rightarrow
f(R_{2s}, R_{1s}, R_{2d}, R_{1d}, d) = \frac{T(R_{2s}, R_{1s}, R_{2d}, R_{1d}, -d)}{L\pi(R_{2d}^2)}
\cdot \frac{N_p}{N_{pole}}
\]

\[
h_j(x) = (x_1, x_2, \ldots, x_n) = 0 \Rightarrow
h_1(R_{2s}, R_{1s}, R_{2d}, R_{1d}) = A_{const} - \pi (R_{2s}^2 - R_{1s}^2 + R_{2d}^2 - R_{1d}^2) = 0
\]

\[
h_2(R_{2s}, R_{1d}, d) = R_{1d} - d - g - R_{2s} = 0
\quad (18)
\]

\[
gi(x_i) = g_i(x_1, x_2, \ldots, x_n) \leq 0 \Rightarrow
\]

\[
g_1(R_{2s}, R_{1s}) = R_{1s} - R_{2s} + 4 \cdot 10^{-3} \leq 0
\]

\[
g_2(R_{2d}, R_{1d}) = R_{1d} - R_{2d} + 4 \cdot 10^{-3} \leq 0
\]

\[
g_4(R_{1s}) = 48.5 \cdot 10^{-3} - R_{1s} \leq 0
\]

\[
g_5(d) = -20 \cdot 10^{-3} + d \leq 0
\]

\[
g_6(d) = 1 \cdot 10^{-3} - d \leq 0.
\quad (19)
\]

A constrained nonlinear-minimization routine was performed to find the optimal dimensions for the magnetic spur gear. The initial design is very close to the computer-optimized solution. The initial design was 141.9 kN·m/m³, and the computer optimized solution was 142.5 kN·m/m³. The reason why the results are so close to each other is mainly caused by tight constrains limitations. The computerized model is locked by an area constrain \( h_1 \), and radius and inner radii must also be within limited values.

The influence of using different amount of magnetic material was also investigated. The investigation has only been made with the 42- and 44-pole configuration and wider boundary on the radius constrains. The result of this analysis is shown in Table II. The optimal torque density is increased to a certain amount if more magnetic material is put into the magnetic cycloid gear.

The last result of the torque-density optimization gave 183 kN · m/m³ or 183 N · m/L, which is nearly twice the capability of state-of-the-art permanent magnetic gears.

## VIII. CONCLUSION

A new cycloid magnetic-gear configuration has been presented. This magnetic gear is characterized by having high torque density and a high gearing ratio. The maximum achievable torque of the proposed gear was calculated by analytical expressions derived in [2] and the transformation equation (16). An initial design of the cycloid gear was able to reach 141.9 N · m/L, which is around 40% more than the “planetary” magnetic gears developed in [4]. An experimental test model of the initial design was constructed and tested. The experimental test model reached 33 N · m. Optimizations for cycloid gears showed that it was possible to reach torque densities up to 183 N · m/L, which is almost twice the density as compared to the “planetary” magnetic-gear types [4]. A cycloid magnetic gear could therefore be a possible choice for future applications where, for example, a motor or generator is integrated together with the cycloid-gear design. The proposed configurations might also be used as power-split devices for hybrid cars or wind turbines with a fixed-speed synchronous generator.

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## REFERENCES


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